Exercise 7C

1 **a** Area of shaded sector

$= \frac{1}{2} \times 8^2 \times 0.6 = 19.2\,\text{cm}^2$

b Area of shaded sector

$= \frac{1}{2} \times 9^2 \times \frac{\pi}{6} = \frac{27\pi}{4} = 6.75\pi\,\text{cm}^2$

c Angle subtended at C by major arc

$= 2\pi - \frac{\pi}{5} = \frac{9\pi}{5}$

Area of shaded sector

$= \frac{1}{2} \times 1.2^2 \times \frac{9\pi}{5} = \frac{162\pi}{125} = 1.296\pi\,\text{cm}^2$

d Area of shaded segment

$= \frac{1}{2} \times 10^2(1.5 - \sin 1.5) = 25.1\,\text{cm}^2$

e Area of shaded segment

$= \frac{1}{2} \times 6^2\left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)$

$= \left(6\pi - 9\sqrt{3}\right)\text{cm}^2 = 3.26\,\text{cm}^2$

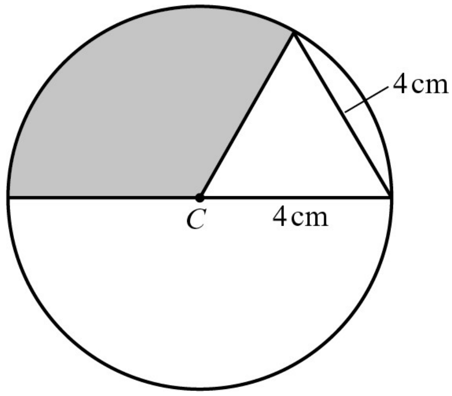
f Area of shaded segment

$= \pi \times 6^2 - \left(\frac{1}{2} \times 6^2\left(\frac{\pi}{4} - \sin\frac{\pi}{4}\right)\right)$

$= 36\pi - \frac{36\pi}{8} + \frac{36}{2} \times \frac{\sqrt{2}}{2}$

$= \left(\frac{63\pi}{2} + 9\sqrt{2}\right)\text{cm}^2 = 111.7\,\text{cm}^2$

2 **a**



The triangle is equilateral, so the angle at C in the triangle is $\frac{\pi}{3}$.

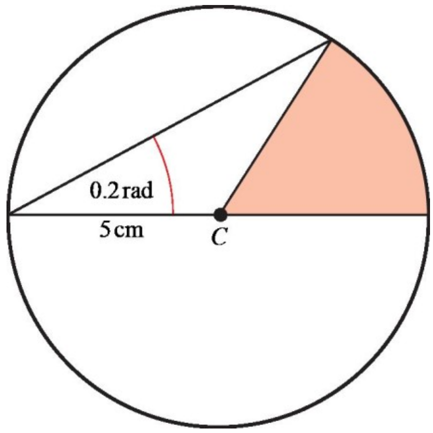
Angle subtended at C by shaded sector

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Area of shaded sector

$= \frac{1}{2} \times 4^2 \times \frac{2\pi}{3} = \frac{16}{3}\pi\,\text{cm}^2$

b

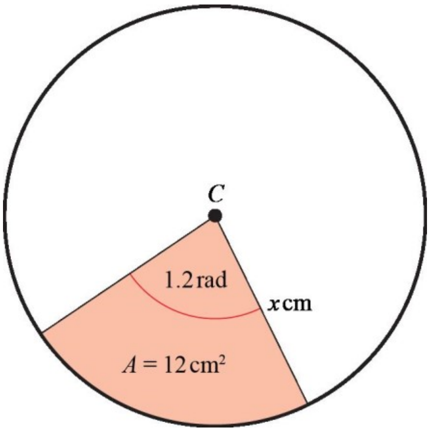


The triangle is isosceles, so the angle at C in the shaded sector is 0.4 rad.

Area of shaded sector

$= \frac{1}{2} \times 5^2 \times 0.4 = 5\,\text{cm}^2$

3 a



Area of shaded sector

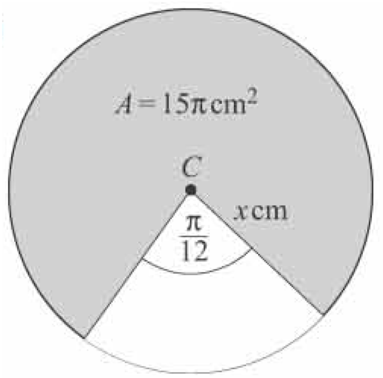
$= \frac{1}{2} \times x^2 \times 1.2 = 0.6x^2$

So $0.6x^2 = 12$

$x^2 = 20$

$x = 4.47$ (3 s.f.)

b



Area of shaded sector

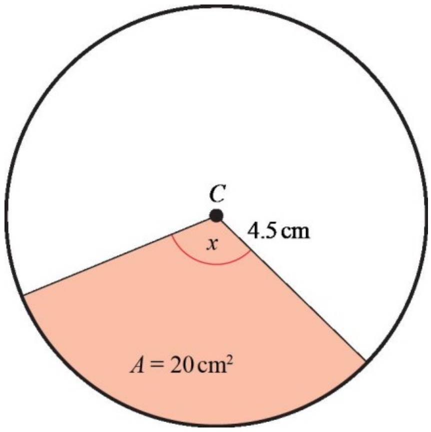
$= \frac{1}{2} \times x^2 \times \left(2\pi - \frac{\pi}{12}\right) = \frac{1}{2} x^2 \times \frac{23\pi}{12}$

So $15\pi = \frac{23}{24}\pi x^2$

$x^2 = \frac{24 \times 15}{23}$

$x = 3.96$ (3 s.f.)

c



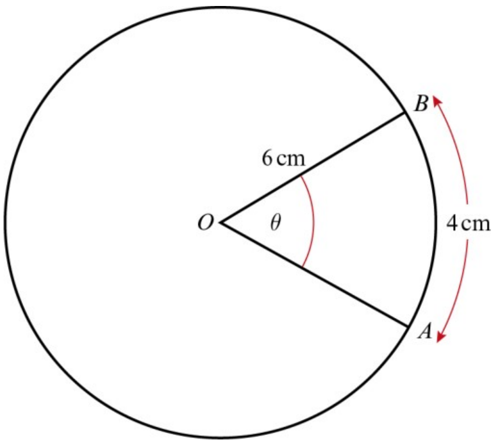
Area of shaded sector

$= \frac{1}{2} \times 4.5^2 \times x$

So $2 = \frac{1}{2} \times 4.5^2 x$

$x = \frac{40}{4.5^2} = 1.98$ (3 s.f.)

4



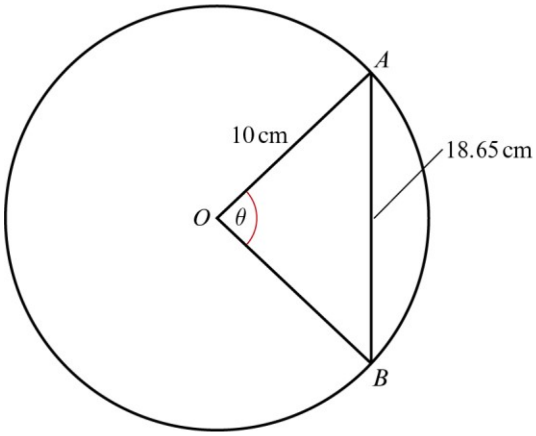
Using $l = r\theta$:

$4 = 6\theta$

$\theta = \frac{2}{3}$

So area of sector $= \frac{1}{2} \times 6^2 \times \frac{2}{3} = 12\text{ cm}^2$

5



a $\cos\theta = \dfrac{10^2 + 10^2 - 18.65^2}{2 \times 10 \times 10}$

$= -0.739$ (3 s.f.)

b $\cos\theta = -0.739... \Rightarrow \theta = 2.4025...$

Area $= \dfrac{1}{2} \times 10^2 \times 2.4025...$

$= 120\,\text{cm}^2$ (3 s.f.)

6 Using area of sector $= \dfrac{1}{2}r^2\theta$:

$100 = \dfrac{1}{2} \times 12^2\theta$

$\Rightarrow \theta = \dfrac{100}{72} = \dfrac{25}{18}$ rad

The perimeter of the sector

$= 12 + 12 + 12\theta = 12(2 + \theta)$

$= 12 \times \dfrac{61}{18} = \dfrac{122}{3} = 40\dfrac{2}{3}$ cm

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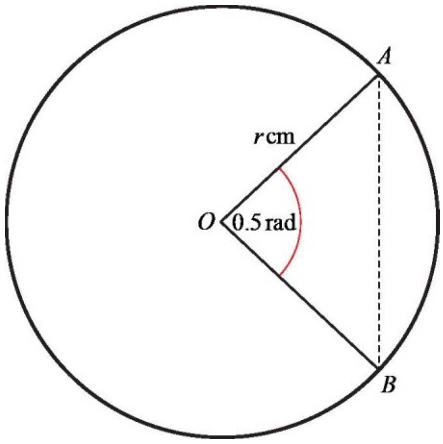


a The perimeter of minor sector AOB

$= r + r + 0.5r = 2.5r$

So $30 = 2.5r$

$\Rightarrow r = \dfrac{30}{2.5} = 12$

b Area of minor sector $AOB = \dfrac{1}{2}r^2\theta$

$= \dfrac{1}{2} \times 12^2 \times 0.5 = 36\,\text{cm}^2$

c Area of segment

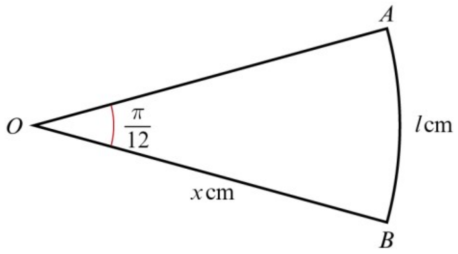
$= \dfrac{1}{2}r^2(\theta - \sin\theta)$

$= \dfrac{1}{2} \times 12^2(0.5 - \sin 0.5)$

$= 72(0.5 - \sin 0.5)$

$= 1.48\,\text{cm}^2$ (3 s.f.)

8



a $l = r\theta \Rightarrow l = x \times \frac{\pi}{12} \Rightarrow x = \frac{12l}{\pi}$

Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times \left(\frac{12l}{\pi}\right)^2 \times \frac{\pi}{12}$

$= \frac{1}{2} \times \frac{12l^2}{\pi}$

$= \frac{6l^2}{\pi}$

b $\frac{6l^2}{\pi} \times 24 = 3600\pi$

$l^2 = 25\pi^2$

$l = 5\pi$

The arc length of AB is 5π cm.

c $x = \frac{12l}{\pi} = \frac{12}{\pi} \times 5\pi = 60$

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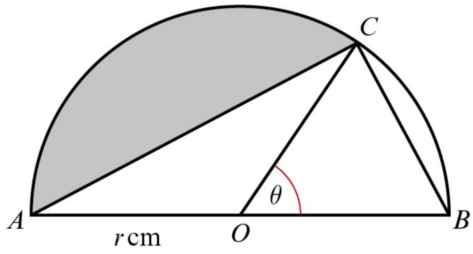

Using the formula,

area of a triangle $= \frac{1}{2}ab\sin C$:

area of triangle $COB = \frac{1}{2}r^2 \sin\theta$ **(1)**

$\angle AOC = \pi - \theta$, so area of shaded segment

$= \frac{1}{2}r^2\left((\pi - \theta) - \sin(\pi - \theta)\right)$ **(2)**

As **(1)** and **(2)** are equal:

$\frac{1}{2}r^2 \sin\theta = \frac{1}{2}r^2\left(\pi - \theta - \sin(\pi - \theta)\right)$

$\sin\theta = \pi - \theta - \sin(\pi - \theta)$

But $\sin(\pi - \theta) = \sin\theta$,

so $\sin\theta = \pi - \theta - \sin\theta$

Hence $\theta + 2\sin\theta = \pi$

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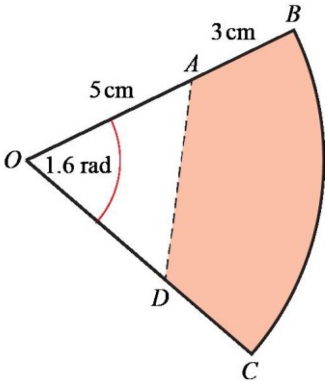


Area of sector $OBC = \frac{1}{2}r^2\theta$

with $r = 8\text{ cm}$ and $\theta = 1.6\text{ rad}$

So area of sector OBC

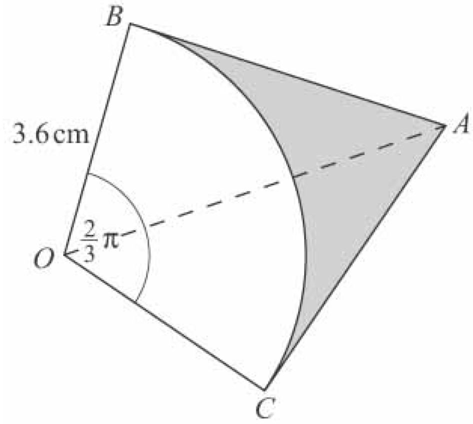
$= \frac{1}{2} \times 8^2 \times 1.6 = 51.2\text{ cm}^2$

Using area of triangle formula:
area of triangle OAD

$= \frac{1}{2} \times 5 \times 5 \times \sin 1.6 = 12.49...\text{cm}^2$

So area of shaded region
$= 51.2 - 12.49... = 38.7\text{ cm}^2$ (3 s.f.)

11



In right-angled triangle OBA :

$\tan\frac{\pi}{3} = \frac{AB}{3.6} \Rightarrow AB = 3.6 \times \tan\frac{\pi}{3}$

So area of triangle OBA

$= \frac{1}{2} \times 3.6 \times 3.6 \times \tan\frac{\pi}{3}$

and area of quadrilateral $OBAC$

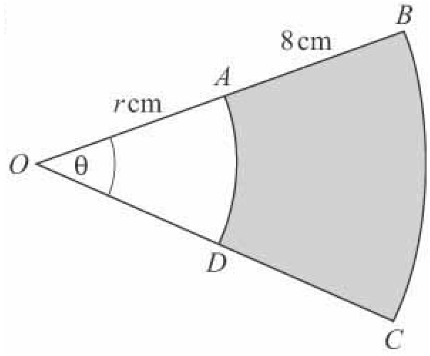
$= 3.6^2 \times \tan\frac{\pi}{3} = 22.447...\text{cm}^2$

Area of sector

$= \frac{1}{2} \times 3.6^2 \times \frac{2}{3}\pi = 13.57...\text{cm}^2$

So area of shaded region
$= 22.447... - 13.57... = 8.88\text{ cm}^2$ (3 s.f.)

12



a Area of sector $OBC = \frac{1}{2}(r+8)^2\,\theta\,\text{cm}^2$

Area of sector $OAD = \frac{1}{2}r^2\theta\,\text{cm}^2$

So area of shaded region $ABCD$

$$= \left(\frac{1}{2}(r+8)^2\theta - \frac{1}{2}r^2\theta\right)\text{cm}^2 = 48\,\text{cm}^2$$

$$\theta\left((r^2+16r+64)-r^2\right) = 96$$

$$\theta(16r+64) = 96$$

$$\theta(r+4) = 6$$

$$r\theta + 4\theta = 6 \qquad \textbf{(1)}$$

$$r\theta = 6 - 4\theta$$

$$r = \frac{6}{\theta} - 4$$

b Substituting $r = 10\theta$ in equation **(1)**:

$6 = 10\theta^2 + 4\theta$

Rearranging:

$5\theta^2 + 2\theta - 3 = 0$

$(5\theta - 3)(\theta + 1) = 0$

So $\theta = \frac{3}{5}$ and $r = 10\times\frac{3}{5} = 6$

Perimeter of shaded region

$= \left(r\theta + 8 + (r+8)\theta + 8\right)\text{cm}$

$= \frac{18}{5} + 8 + \frac{42}{5} + 8 = 28\,\text{cm}$

13 Area of sector $= A\,\text{cm}^2 = \frac{1}{2}\times 28^2\times\theta$

Perimeter of sector $= P\,\text{cm}$

$= r\theta + 2r = (28\theta + 56)\,\text{cm}$

As $A = 4P$:

$392\theta = 4(28\theta + 56)$

$98\theta = 28\theta + 56$

$70\theta = 56$

$\theta = \frac{56}{70} = 0.8$

So $P = 28\theta + 56 = 28\times 0.8 + 56 = 78.4$

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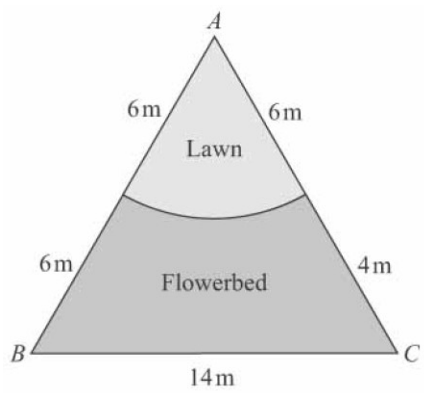


a Using the cosine rule:

$\cos BAC = \frac{10^2 + 12^2 - 14^2}{2\times 10\times 12} = 0.2$

$\angle BAC = \cos^{-1} 0.2$

$= 1.369... = 1.37$ rad (3 s.f.)

b Area of triangle ABC

$= \frac{1}{2}\times 12\times 10\times \sin A = 58.787...\,\text{m}^2$

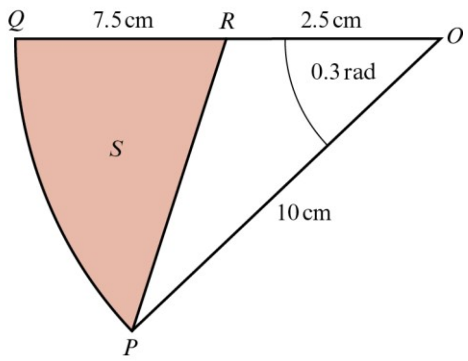
Area of sector (lawn)

$= \frac{1}{2}\times 6^2\times A = 24.649...\,\text{m}^2$

So area of flowerbed

$= 58.787... - 24.649... = 34.1\,\text{m}^2$ (3 s.f.)

15



a $RP^2 = 2.5^2 + 10^2 - 2 \times 10 \times 2.5 \times \cos 0.3$
$= 58.48...$

$RP = 7.65\text{ cm}$

$QP = 10 \times 0.3 = 3\text{ cm}$

So perimeter of S

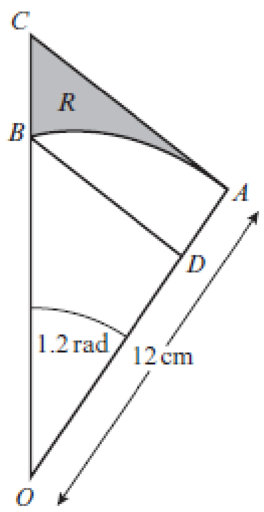
$= 3 + 7.5 + 7.65 = 18.1\text{ cm}$ (3 s.f.)

b Area of S

$= \frac{1}{2} \times 10^2 \times 0.3 - \frac{1}{2} \times 2.5 \times 10 \times \sin 0.3$

$= 11.3\text{ cm}^2$ (3 s.f.)

16 a



$AC = 12 \times \tan 1.2 = 30.865...$ cm

Area of triangle AOC

$= \frac{1}{2} \times 12 \times 30.865... = 185.194...\text{ cm}^2$

So area of R

$= 185.194... - \frac{1}{2} \times 12^2 \times 1.2 = 98.794...$

$= 98.79\text{ cm}^2$ (2 d.p.)

b Length of arc $AB = 12 \times 1.2 = 14.4$ cm

$OD = 12 \times \cos 1.2 = 4.348...\text{ cm}$

$BD = 12 \times \sin 1.2 = 11.184...\text{ cm}$

$AD = 12 - 4.348... = 7.651...\text{ cm}$

Perimeter of DAB

$= AB + AD + BD$

$= 14.4 + 7.651... + 11.184... = 33.236...$

$= 33.24\text{ cm}$ (2 d.p.)

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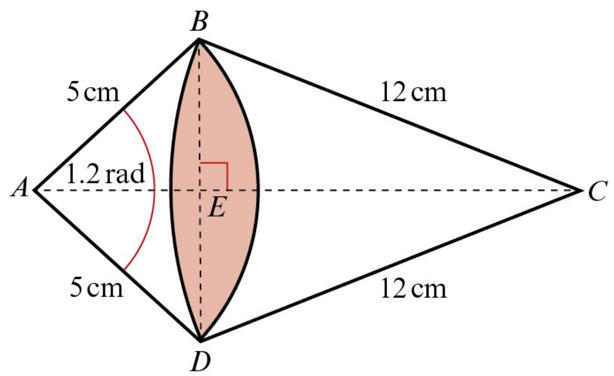

$BE = 5 \times \sin 0.6 = 2.823...$

so $\sin BCE = \frac{2.823...}{12}$

hence $\angle BCE = 0.237...$

and $\angle BCD = 0.474...$

Shaded area to left of BD

$= \frac{1}{2} \times 12^2 \times (0.474... - \sin 0.474...)$

$= 1.271...$

Shaded area to right of BD

$= \frac{1}{2} \times 5^2 \times (1.2 - \sin 1.2)$

$= 3.349...$

So total shaded area

$= 1.271... + 3.349... = 4.62\text{ cm}^2$ (3 s.f.)

Challenge

Arc length $l = r\theta \Rightarrow \theta = \frac{l}{r}$

So area $= \frac{1}{2} r^2\theta = \frac{1}{2} r^2 \left(\frac{l}{r}\right) = \frac{1}{2} rl$